Relativity **Examination** semester 2

TRAINING

Everything must be fully justified on the basis of the definitions or the appendix.

Problem *Cosmological constant*

Part 1: Einstein’s equation

Let’s consider the Einstein’s equation with the cosmological constant Λ: $R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R+\Lambda g_{\mu\nu}=\kappa T_{\mu\nu}$.

1) Demonstrate that the new term satisfy the properties of symmetry and null divergence.

2) Express R as a function of T and the constants Λ and κ.

3) Express $R_{\mu\nu}$ as a function of $T_{\mu\nu}$, T, $g_{\mu\nu}$ and the constants Λ and κ.

4) Give $R_{\mu\nu}$ in vacuum.

If the cosmological constant exists, it is very small. This is linked to dark energy.

Part 2: 3D Spacetime Toy model

There is no gravity in 2D or 3D spacetimes, because then, all the components of the Riemann curvature tensor are null in vacuum. So, since matter cannot curve spacetime in these cases, let’s add a cosmological constant Λ. The following metric is obtained in this theoretical 3D spacetime (with one time coordinate and only two dimensions of space):

$$ds^2=(1+\Lambda r^2)c^2dt^2-\frac{dr^2}{(1+\Lambda r^2)}-r^2d\phi^2, \qquad x^\mu(x^0=ct, x^1=r, x^2=\phi)$$

1) Calculate all the components of the Christoffel symbols.

2) Express all the components of the Riemann curvature tensor for this metric.

3) Then, find the Ricci tensor $R_{\mu\nu}$ components, and the Ricci scalar R.

4) In this spacetime, the Einstein’s equation, in vacuum and with the cosmological constant, give a value for R. Is this the same value as that obtained from the metric in question 3)?

APPENDIX:

Connections (Christoffel symbols): $\Gamma^\alpha{}_{\mu\nu}=\frac{1}{2}g^{\alpha\beta}(\partial_\mu g_{\beta\nu}+\partial_\nu g_{\beta\mu}-\partial_\beta g_{\mu\nu})$

Riemann tensor: $R^\alpha{}_{\beta\gamma\delta}=\Gamma^\alpha{}_{\beta\delta,\gamma}-\Gamma^\alpha{}_{\beta\gamma,\delta}+\Gamma^\alpha{}_{\sigma\gamma}\Gamma^\sigma{}_{\beta\delta}-\Gamma^\alpha{}_{\sigma\delta}\Gamma^\sigma{}_{\beta\gamma}$

Ricci tensor: $R_{\mu\nu}=R^\alpha{}_{\mu\alpha\nu}$

Covariant derivative: $D_\mu A_\nu=\partial_\mu A_\nu-\Gamma^\lambda{}_{\mu\nu}A_\lambda$

Mass-energy scalar: $T=g^{\mu\nu}T_{\mu\nu}$