Accelerating Frame and Rotating Disk

Exercise 1 *Two rockets with the same proper acceleration synchronized*

Equations of the trajectory of an objet in an accelerating frame, at $T=0$, $X=0$ and $Y=0$:

$$X = \frac{1}{\cosh T - \beta_0 \sinh T \cos\theta} - 1 \qquad \text{and} \qquad Y = \frac{\beta_0 \sinh T \sin\theta}{\cosh T - \beta_0 \sinh T \cos\theta},$$

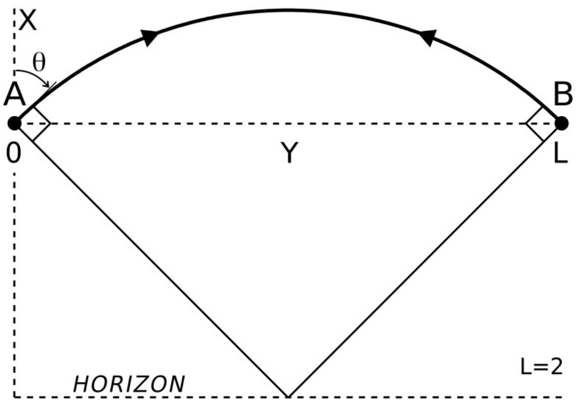
$\beta_0 c$ is the initial speed and θ the angle with the vertical direction. $T=t/t_H$, $X=x/d_H$ and $Y=y/d_H$.

1) Two rockets in the accelerating frame, at rest at *A* and *B*, exchange photons to verify they are well synchronized (see the figure, $X_A=X_B=0$, $Y_A=0$ and $Y_B=2$).

a- How long it takes for a photon to go from *A* to *B*?

b- What distance did the photon travel?

c- What is its average speed?



2) Let's now consider the point of view of an inertial observer in *R'*. At $T'=0$, $T=0$ and $X'=X=0$.

Change of coordinates:

$T'=(X+1)\,shT$, $X'=(X+1)\,chT-1$ and $Y'=Y$.

a- Drawn the two rockets and the ray of light in *R'*.

b- Give the coordinates of the events: the photon starts from *A*, and, the photon arrives at *B*.

c- How long it takes for a photon to go from *A* to *B*?

d- What distance did the photon travel?

e- What is its average speed?

3) Comments.

Exercise 2 *Ehrenfest Paradox*

Metric on the rotating disk: $$ds^2=\left(1-\frac{\rho^2\omega^2}{c^2}\right)c^2dt^2-2\,\rho^2\omega\;dt\;d\theta-d\rho^2-\rho^2d\theta^2-dz^2$$

1) Find the expression of the connections $\Gamma^{\lambda}{}_{\mu\nu}$.

2) Find the components of the Riemann tensor $R^{\alpha}{}_{\beta\mu\nu}$. Comments.

3) Now, let's find the space curvature.

a- When the disk is motionless, give the metric and the surface of the disk of radius *R*.

b- When the disk rotates with an angular speed ω the reference system, has said by *Landau*, is not "synchronous", $g_{0i}\neq 0$, the temporal coordinate is not directly separated from the spatial coordinates. Then, it is shown that the spatial metric is:

$$dl^2=\gamma_{ij}dx^i dx^j \qquad \text{with} \qquad \gamma_{ij}=-g_{ij}+\frac{g_{0i}g_{0j}}{g_{00}}$$

(Landau/Lifchitz, *The Classical Theory of Field*, § Distances and time intervals)

The lengths and the surface $S=\int\sqrt{|det\,\gamma_{ij}|}\,dx^i dx^j$ can be calculate:

i) Express the three-dimensional metric tensor γ_{ij}.

ii) Determine the diameter *D* and the perimeter *P* of the disk.
What is the value of *P/D*? Comments.

iii) Find S and compare with the euclidean surface when $\omega \ll c/R$.